Mon. Not. R. Astron. Soc. **000**, 000–000 (2002)

Fast magnetosonic waves in pulsar winds

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Received/Accepted

ABSTRACT
Fast magnetosonic waves in a magnetically-dominated plasma are investigated. In the pulsar wind, these waves may transport a significant fraction of the energy flux. It is shown that the nonlinear steepening and subsequent formation of multiple shocks is a viable mechanism for the wave dissipation in the pulsar wind. The wave dissipation both in the free pulsar wind and beyond the wind termination shock is considered.

Key words: pulsars:general – supernova remnants – MHD – waves

1 INTRODUCTION

The mechanisms of the energy transfer from the pulsar to the pulsar wind nebula still remain obscure. It is widely accepted that pulsars emit an electron-positron plasma, which form an ultrarelativistic magnetized wind. The rotational energy of the neutron star is carried mostly by electromagnetic fields as Poynting flux (Michel 1982) and should be eventually transferred to the radiating particles. The wind terminates at a shock front located, in the case of the Crab Nebula, some 10^{17} cm from the pulsar. The postshock flow parameters may be matched with the observed Crab structure if the energy flux at the shock front is carried mainly by the particles (Rees & Gunn 1974; Kennel & Coroniti 1984; Emmering & Chevalier 1987). **However it is extremely difficult for the pulsar wind to reach the kinetic energy dominated state, at least if the wind is considered as an ideal MHD flow** (Tomimatsu 1994; Bogovalov 1997; 2001b; Beskin, Kuznetsova & Rafikov 1998; Chiueh, Li & Begelman 1998; Lyubarsky & Eichler 2001). The necessary conversion may be provided by dissipation mechanisms. These mechanisms operate faster at small scales and first affect waves generated in the wind by the rotating, oblique pulsar magnetosphere.

There is a variety of electromagnetic waves in rarefied magnetized plasmas (e.g., Akhiezer et al. 1975) however we assume that only true MHD waves (those satisfying the condition $\mathbf{E} = \mathbf{v} \times \mathbf{B}$) may be generated by the rotating magnetosphere. The reason is that according to the convenient view, the plasma density in the pulsar wind is sufficiently large such that **low-frequency** electromagnetic waves are heavily damped (e.g., Asseo et al. 1978; Melatos & Melrose 1996). There are four types of MHD waves but only the entropy and the fast magnetosonic (FMS) waves may propagate far beyond the light cylinder where the magnetic field is predominantly toroidal. **In an oblique magnetosphere, the magnetic field at the light cylinder oscillates with the pulsar rotation period. These oscillations propagate outwards as MHD waves. In the equatorial belt of the flow, the magnetic field line at a given radius alternates in direction with the frequency of rotation, being connected to a different magnetic pole every half-period. Such an alternating field is transported by the flow in the form of the striped wind (Michel 1971, 1982; Bogovalov 1999), which may be considered as an entropy wave. At high latitudes, where the magnetic field does not change sign, the magnetic oscillations are transferred away in the form of FMS waves (generation of FMS waves by the rotating, slightly nonaxisymmetric magnetosphere was considered by Bogovalov 2001a)**.

The entropy wave decays because of the current starvation in current sheets separating strips with the opposite magnetic field (Usov 1975; Michel 1982, 1994; Coroniti 1990). Lyubarsky & Kirk (2001) showed that the flow significantly accelerates in the course of reconnection and this dilates the timescale over which the wave decays. At typical conditions, the dissipation radius exceeds the radius of the termination shock therefore one should conclude that the Poynting flux in the striped wind does not dissipate until the wind enters the termination shock. All the energy should release within the shock where the flow decelerates.

In this article, the fate of FMS waves is considered. It will be shown that these waves may be described within the MHD framework throughout the pulsar wind up to the termination shock and ever beyond. The reason is that in a magnetically dominated plasma, the FMS waves excite small conductivity currents, the oscillations of the magnetic field being nearly compensated by the displacement current. In MHD regime, FMS waves may decay due to the nonlinear steepening and subsequent formation of multiple shocks. It will be shown that for typical pulsar parameters, the waves may decay only beyond the termination shock. However in rapidly spinning pulsars, like the Crab, FMS waves may decay before the flow reaches the termination shock provided the plasma density in the wind is high enough.

The article is organized as follows. Properties of FMS waves in a magnetically dominated plasma are outlined qualitatively in Sect. 2. These estimates are applied to the pulsar wind in Sect. 3. The wave decay after the multiple shocks formation is considered in Sect. 4. Conclusions are summarized in Sect. 5. In Appendix 1, exact solutions for nonlinear FMS waves in the magnetically dominated plasma are presented. In Appendix 2, the energy and momentum fluxes transferred by FMS waves are derived.

2 FMS WAVES IN A MAGNETICALLY DOMINATED PLASMA

In this section, I consider FMS waves in the wind frame. Let us consider waves propagating perpendicularly to the magnetic field because far enough from the pulsar, the magnetic field is nearly toroidal whereas the waves propagate radially. Only qualitative estimates are outlined here; the exact solutions are presented in Appendix 1.

When the FMS wave propagates perpendicularly to the magnetic field, plasma oscillates along the wave direction together with the magnetic field. The flux freezing condition ties the plasma density and the magnetic field together:

$$\frac{B'}{n}=\frac{B'_*}{n_*}\equiv b=const. \tag{1}$$

Here B' and n are the magnetic field and the plasma density in the proper plasma frame, **the subscript * stands for quantities averaged over the wave period. The quantities measured in the proper plasma frame differ from those measured in the wind frame because the plasma oscillates. In this section, I neglect this difference assuming the oscillation velocity to be nonrelativistic. More general consideration is presented in Appendix 1.** With the aid of Eq.(1), one can express the magnetic energy and the pressure via the plasma density thus considering the wave as the sound wave in a medium with the equation of state

$$\mathcal{P}=p+\frac{b^2n^2}{8\pi};\qquad \mathcal{E}=\varepsilon+\frac{b^2n^2}{8\pi},$$

where p and ε are the plasma pressure and energy density, correspondingly. The velocity of the wave may be now found immediately as the sound velocity in such a medium(throughout the paper all velocities are expressed in units of the speed of light, $c=1$) :

$$s_M^2=\frac{d\mathcal{P}}{d\mathcal{E}}=\frac{ws^2+b^2n^2/4\pi}{w+b^2n^2/4\pi}, \tag{2}$$

where $w=p+\varepsilon$ is the enthalpy of the plasma, $s=\sqrt{dp/d\varepsilon}$ the thermal sound velocity.

In the pulsar wind, the magnetic energy density significantly exceeds the plasma energy density. One can conveniently express all the values via the parameter

$$\sigma=\frac{b^2n^2}{4\pi w}, \tag{3}$$

which is twice the ratio of the magnetic to the plasma energy density in the wind frame and the ratio of the Poynting flux to the plasma energy flux in the laboratory frame. One can see that at $\sigma\gg 1$ the FMS velocity is close to the speed of light. The corresponding Lorentz factor, $\gamma_M\equiv(1-s_M^2)^{-1/2}$, may be written as

$$\gamma_M=\sqrt{\frac{\sigma}{1-s^2}}.$$

In the small amplitude, linear wave, the electric field may be written as $E'=vB'_*$ whereas variations in the magnetic field are $\delta B'=B'_*(\delta n/n_*)$. Taking into account that variations in the density and in the velocity are related by the standard expression $\delta n/n_*=v/s_M$, one can see that

$$E'=\frac{\delta n}{n_*}=\delta B' \tag{4}$$

to within a factor of $1/\sigma$. Because the electric field may not exceed the magnetic field, the wave amplitude should be limited by

$$\delta B'<B'_*/2. \tag{5}$$

It is shown in the Appendix 1 that when the wave amplitude approaches the limiting value, the oscillation velocity becomes ultrarelativistic, the plasma density goes to zero at some phase of the wave period and moreover the characteristic scale for the nonlinear steepening of the wave decreases to zero.

Now let us find what plasma density is necessary to allow the MHD solution. The plasma density should satisfy the evident condition $en>j$, where j is the conductivity current excited in the wave. Substituting into the Maxwell equations

$$\nabla\times\mathbf{B}'=4\pi\mathbf{j}+\frac{\partial\mathbf{E}'}{\partial t};\qquad \nabla\times\mathbf{E}'=-\frac{\partial\mathbf{B}'}{\partial t},$$

a sinusoidal wave $\exp(-i\omega t+\mathbf{k}\cdot\mathbf{x})$ with the dispersion law $\omega/k=s_M$, one can find the conductivity current in this wave,

$$j=(1-s_M^2)i\omega E'/4\pi.$$

One can see that in high-σ flows, the conductivity current in FMS waves is σ times less than the displacement current. Substituting the maximal current density, $j=en$, and making use of Eqs.(2–4), one can see that the cold plasma may provide the necessary conductivity current if

$$\omega_B>\omega\frac{\delta B'}{B'_*}, \tag{6}$$

where $\omega_B=eB'_*/m$ is the Larmour frequency. We consider the electron-positron plasma therefore throughout the paper m is the electron mass.

At the conditions (5, 6), FMS waves in a magnetically dominated plasma may be considered in MHD approximation. In the MHD regime, the wave may decay as a result of nonlinear steepening and subsequent formation of multiple shocks. The wave steepening occurs because the wave velocity depends on the plasma density and therefore the compressive part of the wave moves faster than the expansive one. In the strongly magnetized plasma, even a large amplitude wave is only weakly nonlinear because the wave velocity varies with the density only by a factor of about $1/\sigma$. The reason is that, as it was shown above, the conductivity current, which only introduces nonlinearity into the MHD equations, is small in the magnetically dominated plasma.

The difference in the velocities between the compressive and expansive parts of the wave may be estimated from Eq.(2) as

$$\delta s_M \sim \frac{1}{\sigma}\frac{\delta n}{n_*}.$$

The shock forms when the leading edge of the wavefront becomes vertical (e.g., Landau & Lifshitz 1959). This occurs within a characteristic nonlinear time it takes from the compressive part to shift relative the expansive one by $\sim 1/\omega$. Therefore the shock forms after the wave travels a distance

$$x_{\rm nl} \sim 1/(\delta s_M \omega) \sim \sigma/\omega. \qquad (7)$$

This estimate is confirmed by a rigorous derivation in Appendix 1. After the shock formation, the wave continue to distort at the characteristic scale $x_{\rm nl}$ such that eventually all the wave energy dissipates in the shock. However both σ and ω may change because the plasma heats up and accelerates (or decelerates) considerably in the course of the wave dissipation. Therefore numerically the dissipation scale may differ from the shock formation scale even though the same expression (7) is valid for both scales.

3 FMS WAVES IN THE PULSAR WIND

Far beyond the light cylinder, the wind may be considered as purely radial, whereas the magnetic field as purely toroidal. The wind is assumed to be super-FMS, $\gamma > \sqrt{\sigma}$. Transforming the electromagnetic fields from the wind frame into the pulsar frame, one can see that the condition (5) reduces to the condition $\delta B < B_*$ (the factor of two appears because in the proper plasma frame $E'_* = 0$ whereas in the pulsar frame $E_* = v_* B_* \approx B_*$). So FMS waves may be generated by the rotating oblique magnetosphere at high latitudes where the magnetic field does not change the sign. In the equatorial belt (its width depends on the angle between the magnetic and the rotation axes of the pulsar) an entropy wave is generated in the form of the striped wind. Generally a superposition of an entropy and FMS waves is generated here (one can talk about the superposition because even a large amplitude FMS wave is weakly nonlinear in the magnetically dominated plasma) however an important point is that an entropy wave arises here inevitably because there is no other MHD wave that may transfer alternating magnetic field in a high-σ plasma.

Now let us consider validity of MHD approximation (Eq.(6)) for the FMS wave. The magnetic field in the pulsar wind is predominantly toroidal and may be presented as **(from now and hereafter only averaged quantities will be considered therefore the subscript * will be omitted)**

$$B = B_L \frac{R_L}{R}, \qquad (8)$$

where $R_L = c/\Omega = 5 \cdot 10^9 P$ cm is the light cylinder radius, P the pulsar period. The magnetic field at the light cylinder may be estimated as

$$B_L = \frac{\mu}{R_L^3} = 9\frac{\mu_{30}}{P^3}\ \mathrm{G},$$

where $\mu = 10^{30}\mu_{30}$ G·cm^3 is the magnetic moment of the star. The wave frequency in the pulsar frame is the pulsar rotation frequency; in the wind frame the frequency is $\omega = \Omega/(2\gamma)$. Noting that the magnetic field in the wind frame is $B' = B/\gamma$, one can reduce the condition (6) to

$$R < 5 \cdot 10^{17}\frac{\mu_{30}}{P}\ \mathrm{cm}. \qquad (9)$$

This radius should be compared with the radius of the standing shock where the wind terminates. One can place the upper limit on this radius by equating the magnetic pressure in the wind to the ram pressure of the medium, $B^2/8\pi = \rho V^2$, where $\rho = 1\rho_0$ g/cm^3 is the density of the interstellar gas, $V = 100V_2$ km/s the velocity of pulsar through it. Substituting Eq.(8), one obtains

$$R_{\rm shock} = 7 \times 10^{14}\frac{\mu_{30}}{V_2\sqrt{\rho_0}P^2}\ \mathrm{cm} \qquad (10)$$

The radius of the terminating shock may be less than (10) if the pulsar is surrounded by a dense plerion. Comparing Eq.(10) with Eq.(9) one can see that FMS waves in the pulsar wind may be considered in MHD approximation.

The wave may decay in multiple shocks that arise as a result of nonlinear steepening. Transforming the characteristic nonlinear scale (7) to the pulsar frame, one gets

$$R_{\rm nl} = 4\gamma^2\sigma R_L. \qquad (11)$$

The magnetization parameter of the wind depends on the plasma density. In the pulsar frame, one can conveniently express the density, $N = n\gamma$, via the dimensionless multiplicity factor, κ, as

$$N = \frac{\kappa\Omega B_L}{2\pi e}\left(\frac{R_L}{R}\right)^2. \qquad (12)$$

Before the multiple shocks arise, the pulsar wind is cold, $w = mn$, and propagates with a constant Lorentz factor γ_0. The magnetization parameter remains constant,

$$\sigma_0 = \frac{B^2}{4\pi m N\gamma_0} = \frac{\omega_L}{2\kappa\gamma_0\Omega},$$

where $\omega_L \equiv eB_L/m$ is the gyrofrequency at the light cylinder. Now one can write the shock formation distance as

$$R_0 = 4\gamma_0^2\sigma_0 R_L = \frac{2\gamma_0\omega_L}{\kappa\Omega^2} = 2.5 \times 10^{17}\frac{\gamma_0\mu_{30}}{\kappa P}\ \mathrm{cm}. \qquad (13)$$

For typical pulsar parameters, $\gamma_0 \sim 100$, $\kappa \sim 1 - 10^3$ (Hibschmann & Arons 2001), this radius satisfies the condition (9) therefore MHD description of the wave is justified. For the typical parameters, this radius exceeds the upper limit on the termination shock radius (10) for all pulsars with the exception of the millisecond ones. However there is strong evidence to suggest that the multiplicity parameter in the Crab pulsar significantly exceeds the typical one and may be as high as $\kappa \sim 10^6$ (Shklovsky 1970; Rees & Gunn 1974). Extra pairs may be produced together with the observed gamma radiation in the upper magnetosphere (Cheng, Ho & Ruderman 1986; Lyubarskii 1996). At such a plasma density, the multiple shocks arise before the flow reaches the termination shock (observed in the Crab at $\sim 10^{17}$ cm from the pulsar). So formation of the shocks in the FMS waves may occur both in the free pulsar wind within the termination shock and in the pulsar wind nebula beyond the termination shock.

If the multiple shocks are not formed until the flow reaches the termination shock (i.e., if the radius (13) exceeds the termination shock radius), the waves enter the nebula through the shock. In case the upstream flow is not Poynting dominated, the downstream flow is non-relativistic; the nonlinear scale (11) is extremely small in this case and therefore the waves should decay immediately, in fact already within the shock. But if the upstream flow is still Poynting dominated, the shock is weak and the downstream flow is relativistic (Kundt & Krotschek 1982, Kennel & Coroniti 1984). The waves pass freely the shock from upstream because reflection is impossible. Parameters of the waves are nearly not affected by weak shocks (e.g., Anderson 1963) however the nonlinear scale decreases because the flow decelerates (the statement by Kennel & Coroniti that the downstream Lorentz factor remains large, $\sim \sqrt{\sigma}$, is based on the assumption that the flow is spherically symmetric; one can see that even slight deviation of the flow lines from radial may result in significant deceleration of the flow). Eventually the multiple shocks are formed. Investigation of the flow in the nebula and search for the multiple shocks formation zone is out of the scope of this article. I simply assume that the multiple shocks do form at some point and consider evolution of the flow parameters in the course of the wave dissipation. It will be shown in the next section that in the sub-FMS flow (beyond the termination shock) the wave energy dissipates immediately after the multiple shocks arise, dissipation being accompanied by the flow deceleration down to $\gamma \sim 1$. Hence although the Poynting dominated wind could not be brought to rest in the termination shock, the wave dissipation provides an effective mechanism of the flow braking.

4 WAVE DECAY

4.1 Basic equations

After the multiple shocks arise, the wave energy dissipates. This occurs at the nonlinear scale (11), which now may change because the released energy heats the plasma flow. One can consider evolution of the flow parameters in the course of the wave decay applying the conservation laws to the system. Let us separate the average fluxes of conserving values into the wave and the flow parts. Such a separation may be performed in general terms for small amplitude waves (see Appendix 2). The average plasma density, n, and velocity, v, (and the corresponding Lorentz factor γ) are defined such that the wave does not contribute to the matter flux (in Appendix 2 the averaged quantities are marked by tilde; here we work only with averaged quantities and tilde is omitted). Therefore the continuity equation may be written as (the flow is considered as locally spherically symmetric)

$$n\gamma v R^2 = n_0\gamma_0 v_0 R_0^2, \tag{14}$$

where the subscript "0" is referred to the quantities at the shock formation radius (13). The energy equation may be written as

$$\frac{1}{R^2}\frac{d}{dR}\left(w\gamma^2 vR^2 + \frac{B^2}{4\pi}vR^2\right) = Q, \tag{15}$$

where Q is the energy transferred from the wave to unit plasma volume per unit time. The magnetic field strength may be expressed via the density making use of the frozen flux condition (see Eq.(A1.16))

$$\frac{B}{Rn\gamma} = \frac{B_0}{R_0 n_0 \gamma_0}, \tag{16}$$

The entropy equation describes the entropy growth as a result of the wave decay:

$$T\frac{dS}{d\tau} = p\frac{d}{d\tau}\frac{1}{n} + 3\frac{dT}{d\tau}, \tag{17}$$

where $d\tau = dR/\gamma$ is the proper time. We assume that just after the shock formation, the plasma is heated to relativistic temperatures, $w = 4nT$.

Because the wave propagates in the proper plasma frame with the velocity $s_M = 1 - O(1/\sigma)$, its energy and momentum are equal, in this frame, to within $1/\sigma$ (see Appendix 2). Therefore if the energy $d\varepsilon$ per unit mass is absorbed in the proper plasma frame, then the momentum $d\varepsilon$ per unit mass is also transferred from the wave to the plasma. Then in the pulsar frame, the absorbed energy is $dE = 2\gamma d\varepsilon$. Taking into account that $dVdt$ is invariant, one can write the energy release rate as

$$Q \equiv \frac{dE}{dVdt} = 2\gamma n\frac{d\varepsilon}{d\tau}.$$

Substituting $d\varepsilon = TdS$, one writes the entropy equation in the form

$$2\gamma^2\left(3n\frac{dT}{dR} - T\frac{dn}{dR}\right) = Q. \tag{18}$$

Let us now turn to the energy equation (15). Making use of Eqs.(14, 16), one can see that the Poynting flux (the second term in the brackets in the left-hand side) is nearly independent of R in the ultrarelativistic case. Expanding this term in $1/\gamma$ to the next order and retaining in the first term only leading terms in $1/\gamma$ because this term is already small as $1/\sigma$, one can reduce Eq.(15) to

$$\frac{d}{dR}\left(w\gamma^2 R^2 + \frac{B_0^2 R_0^2}{8\pi\gamma^2}R^2\right) = QR^2, \tag{19}$$

Substituting n from Eq.(14) into Eqs.(18, 19), one gets finally

$$\frac{d}{dR}\left(\frac{T\gamma}{T_0\gamma_0} + \frac{a\gamma_0^2}{3\gamma^2}\right) = \frac{QR^2}{4T_0\gamma_0^2 n_0 R_0^2}; \tag{20}$$

$$3\frac{\gamma}{\gamma_0}\frac{d(T/T_0)}{dR} + \frac{T}{T_0}\frac{d(\gamma/\gamma_0)}{dR} + \frac{2T\gamma}{T_0\gamma_0 R} = \frac{QR^2}{2T_0 n_0\gamma_0^2 R_0^2}; \tag{21}$$

where the factor

$$a = \frac{3B_0^2}{32\pi n_0 T_0\gamma_0^2} = \frac{3\sigma_0}{2\gamma_0^2}. \tag{22}$$

is less than unity in super-FMS flows and exceeds unity in the opposite case. It was found above that at different wind parameters, the multiple shocks may arise in FMS waves both before the wind reaches the termination shock and beyond the shock in the pulsar wind nebula. Therefore one should consider the wave decay both in the super-FMS (free wind) and in the sub-FMS (beyond the termination shock) flows. The flow behavior in these cases is just opposite therefore let us consider them separately.

4.2 Wave decay in the super-FMS flow

The flow in the wind is super-FMS, in this case $a \ll 1$ and the plasma is cold at R_0 so one can take $T_0 \sim m$. Eliminating Q from Eqs.(20, 21) and neglecting the term with a, one gets

$$\frac{\gamma}{\gamma_0}\frac{d(T/T_0)}{dR} - \frac{T}{T_0}\frac{d(\gamma/\gamma_0)}{dR} + \frac{2T\gamma}{T_0\gamma_0 R} = 0,$$

which yields

$$\frac{TR^2}{\gamma} = \frac{T_0R_0^2}{\gamma_0}. \tag{23}$$

So in the super-FMS flow heating of the medium is accompanied by acceleration.

The energy release is caused by the nonlinear wave distortion. Therefore the fraction of the wave energy dissipated at the distance ΔR may be roughly estimated as $\Delta R/R_{\rm nl}$. One should substitute in the expression for the nonlinear scale (11) parameters varying in the course of the energy dissipation. Variation of σ may be found making use of Eqs.(14, 16, 23):

$$\sigma \equiv \frac{B^2}{16\pi nT\gamma} = \sigma_0\left(\frac{\gamma_0 R}{\gamma R_0}\right)^2.$$

Now the dissipated fraction of the wave energy may be estimated as $\sim \Delta R/(\sigma\gamma^2 R_L) = R_0\Delta R/R^2$. One can see that the significant part of the wave energy dissipates at the scale $\Delta R \sim R_0$. The dissipation scale is roughly equal to the shock formation scale because acceleration of the flow is compensated by the plasma heating and corresponding decreasing of σ.

Taking into account that the wave energy initially exceeds the plasma energy about σ_0 times (for large amplitude waves), one can see from Eq.(20) (at $a \ll 1$) that $T\gamma/(T_0\gamma_0) \sim \sigma_0$ after the wave dissipates. This implies, with the aid of Eq.(23), that $T \sim m\sqrt{\sigma_0}(R_0/R)$, $\gamma \sim \sqrt{\sigma_0}\gamma_0(R/R_0)$. So the wave dissipation at $R \sim R_0$ heats the plasma till the temperature $\sim m\sqrt{\sigma_0}$ and accelerates it to the Lorentz factor $\sim \sqrt{\sigma_0}\gamma_0$. Then the heated plasma cools and accelerates in the Bernoulli regime, $T \propto 1/R$, $\gamma \propto R$. At the distance $R_1 \sim \sqrt{\sigma_0}R_0$, all the released energy transforms to the kinetic energy and the plasma reaches the Lorentz factor

$$\gamma_1 \sim \sigma_0\gamma_0 = \frac{\omega_L}{2\kappa\Omega} = 1.3\times 10^7\frac{\mu_{30}}{\kappa P^2}.$$

4.3 Wave decay in the sub-FMS flow

If the multiple shocks formation radius (13) exceeds the termination shock radius the waves pass the termination shock and enter the sub-FMS flow. In the sub-FMS case, the flow decelerates in the course of energy release (see below) and the nonlinear scale sharply decreases just after the multiple shocks formation. Therefore the wave decay scale turns out to be small as compared with the radius and one can consider the flow in the plane geometry. In this case Eqs.(20, 21) reduce to

$$\frac{d}{dx}\left(\frac{T\gamma}{T_0\gamma_0} + \frac{a\gamma_0^2}{3\gamma^2}\right) = 1; \tag{24}$$

$$3\frac{\gamma}{\gamma_0}\frac{d(T/T_0)}{dx} + \frac{T}{T_0}\frac{d(\gamma/\gamma_0)}{dx} = 2, \tag{25}$$

where

$$x = \int_{r_0}^{r}\frac{Q}{4n_0T_0\gamma_0^2}dR$$

is the ratio of the transferred energy to the initial plasma energy. When the wave decays, x becomes very large and reaches the initial ratio of the wave energy to the plasma energy; for large amplitude waves this ratio is about σ_0.

Integrating Eq.(24), one yields

$$\frac{T\gamma}{T_0\gamma_0} = 1 + \frac{a}{3} + x - \frac{a\gamma_0^2}{3\gamma^2}.$$

Substituting T from this relation into Eq.(25), one gets the equation

$$\frac{2}{\gamma}\left(1 + \frac{a}{3} + x - \frac{4a\gamma_0^2}{3\gamma^2}\right)\frac{d\gamma}{dx} = 1,$$

which is linear with respect to $x(\gamma)$. The solution is

$$\frac{\gamma^2}{\gamma_0^2} = \frac{3 + a + 3x \pm \sqrt{9(1-a)^2 + 6x(3+a) + 9x^2}}{2(3-a)}, \tag{26}$$

where the sign of the square root should be positive for $a < 1$ and, correspondingly, negative for $a > 1$ to satisfy the initial conditions. One can see that the super-FMS flow ($a < 1$) accelerates whereas the sub-FMS flow ($a > 1$) decelerates in the course of energy release. For $a > 1$, $x \gg 1$ one finds, with the aid of Eq.(22),

$$\gamma = \gamma_0\sqrt{\frac{2a}{3x}} = \sqrt{\frac{\sigma_0}{x}}; \qquad T = T_0\sqrt{\frac{3x^3}{8a}}. \tag{27}$$

It follows from Eqs.(14, 16) that in the narrow decay zone, $R - R_0 \ll R_0$, the magnetic field remains constant while the flow remains relativistic, $v \approx 1$. The absorbed energy is transformed into the internal plasma energy and the magnetization parameter decreases:

$$\sigma \equiv \frac{B^2}{4\pi w\gamma^2} = \frac{2\sigma_0}{x}.$$

The wave energy dissipates completely when $x \sim \sigma_0 \gg 1$. The nonlinear scale (11) decreases already when x exceeds unity i.e. when the fraction of the dissipated energy is small ($\sim 1/\sigma_0$) therefore the total dissipation scale turns out to be small, $\sim R_0/\sigma_0$.

When the wave decays completely, $x \sim \sigma_0$ and one gets $\gamma \sim 1$, $\sigma \sim 1$. However this solution becomes invalid when, in the proper plasma frame, the particle Larmour radius, $r_g \equiv T/(eB')$, exceeds the wavelength, $\lambda = 2\pi\gamma R_L$. Making use of Eqs.(8, 12, 22, 27), one can write the condition $r_g \sim 2\pi\gamma R_L$ as

$$\frac{x}{\sigma_0} \sim \left(\frac{32\pi\kappa R_L}{R_0}\right)^{2/3}.$$

The right-hand side of this expression is small therefore only a small fraction of the energy is transferred to the plasma in MHD regime; most of the energy is dissipated when the Larmour radius exceeds the wavelength. This may result in formation of a high energy tail in the particle energy distribution.

5 CONCLUSIONS

In the pulsar wind, a significant fraction of the Poynting flux may be transported by FMS waves. The plasma in the pulsar wind is magnetically dominated in the sense that in the proper plasma frame, the magnetic field energy density exceeds the plasma energy density. This condition is equivalent to the condition that in the pulsar frame, the Poynting flux dominates the plasma energy flux. In such a plasma, FMS waves excite small conductivity current, oscillations of the magnetic field being nearly compensated by the displacement current. Thus a very small plasma density is sufficient to keep this current and MHD description of these waves is justified throughout the pulsar wind till the termination shock and beyond.

These waves may decay in multiple shocks that arise through nonlinear steepening of the waves. Because the conductivity current is small at the pulsar wind conditions, even a large amplitude FMS wave is nearly linear and therefore shocks are formed at large distances from the pulsar. **Depending on the plasma density in the wind, this may occur either before the flow reaches the termination shock or beyond the shock. In any case the wave energy adequately dissipates in the multiple shocks. Energy release in the super-FMS flow is accompanied by the plasma heating and acceleration. The thermal pressure also does work on the flow and the plasma accelerates further on. Therefore if the waves dissipate in the free, super-FMS pulsar wind, all the wave energy eventually transforms into the kinetic energy of the flow. If the waves do not dissipate in the wind, the flow remains Poynting dominated. In this case the termination shock should be weak and the downstream flow remains ultra-relativistic. Such a flow does not match the slow expansion of the nebula; this was considered as an evidence for conversion of a significant fraction of the Poynting flux into the particle energy flux before the pulsar wind reaches the termination shock (Rees & Gunn 1974; Kennel & Coroniti 1984). However it was shown above that the wave dissipation in the sub-FMS downstream flow is accompanied by the flow deceleration to subrelativistic velocities. Hence even if the flow remains Pointing dominated at the termination shock, the wave dissipation downstream the shock may provide the necessary deceleration.**

Only a fraction of the total Poynting flux is transferred by the FMS waves therefore the proposed mechanism does not solve the σ-problem but should be considered as an element of the future complete theory. The mechanism considered may also play an essential role in gamma-ray burst models involving Poynting dominated outflows from compact objects (Usov 1992, 1994; Mészáros & Rees 1997; Kluzniak & Ruderman 1997; Blackman & Yi 1998; Spruit 1999; Lyutikov & Blackman 2001; Drenkhahn 2002; Drenkhahn & Spruit 2002).

Acknowledgments

I am grateful to John Kirk who initiated this research for kind hospitality during my stay in MPI für Kernphysik. The work was supported by MPI für Kernphysik under their international visitor program and by Ben Gurion University through a seed grant.

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Appendix 1. Nonlinear FMS waves in a magnetically dominated plasma

Relativistic nonlinear MHD waves are considered in the general case by Akhiezer et al (1975). Here we consider only the waves propagating perpendicularly to the ambient magnetic field in the case $\sigma \gg 1$.

Let us first consider the waves in the plane geometry. It is followed from the continuity equation

$$\frac{\partial}{\partial t}\gamma n + \frac{\partial}{\partial x}\gamma n v = 0 \qquad (A1.1)$$

and the frozen-in condition

$$\frac{\partial B}{\partial t}+\frac{\partial}{\partial}vB=0$$

that the magnetic field may be presented in the form

$$B=bn\gamma,\qquad b\equiv\frac{B_*}{n_*\gamma_*}=const,\qquad (A1.2)$$

where the subscript * stands for quantities at some fiducial point; one can conveniently choose this point in such a way that B_* is equal to B averaged over the wave period. The dynamic equation may be presented in the form of the energy equation

$$\frac{\partial T_{00}}{\partial t}+\frac{\partial T_{01}}{\partial x}=0,\qquad (A1.3)$$

where the components of the energy-momentum tensor are

$$T_{01}=w\gamma^2v+\frac{B^2}{4\pi}\gamma^2v;\qquad (A1.4)$$

$$T_{00}=w\gamma^2-p+\frac{1+v^2}{8\pi}B^2\gamma^2.\qquad (A1.5)$$

Note that this set of equations is reduced, with the aid of Eq.(A1.2), to the standard hydrodynamics equations with

$$\mathcal{E}=\varepsilon+\frac{B^2}{8\pi};\quad \mathcal{P}=p+\frac{B^2}{8\pi}=Kn^\Gamma+\frac{b^2n^2}{8\pi};$$

$$\mathcal{W}=\mathcal{E}+\mathcal{P}=\varepsilon+p+\frac{B^2}{4\pi}.\qquad (A1.6)$$

The nonlinear simple wave may be found from the condition that all dependent variables are functions of one of them, e.g., n (Landau & Lifshitz 1959). This means that Eq.(A1.3) should be equivalent to Eq.(A1.1), i.e.

$$\frac{dT_{01}}{dT_{00}}=\frac{d(n\gamma v)}{d(n\gamma)}.\qquad (A1.7)$$

The last equation reduces to

$$\gamma^2\frac{dv}{dn}=\frac{s_M}{n},\qquad (A1.8)$$

where the FMS velocity s_M is given by Eq.(2). For the small wave amplitude, $\delta n\equiv n-n_*\ll n_*$, one obtains the linear FMS wave propagating with the phase velocity $s_M=const$. For a strong wave, s_M depends on the local density therefore the wave becomes nonlinear. However in the strongly magnetized case, $\sigma\gg 1$, s_M goes to unity and therefore even a strong wave is nearly linear.

Substituting $s_M=1$ into Eq.(A1.8), one obtains

$$\frac{n}{n_*}=\sqrt{\frac{1+v}{1-v}\frac{1-v_*}{1+v_*}}.\qquad (A1.9)$$

In the frame moving, in average, with the plasma, $v_*=0$, one gets for the small amplitude wave

$$v=\delta n/n_*.$$

In the case $\gamma\gg 1$ Eq.(A1.9) reduces to

$$\gamma=\gamma_* n/n_*.$$

The waveform moves along the characteristic of Eq.(A1.1):

$$\frac{dx}{dt}=\frac{d(n\gamma v)}{d(n\gamma)}=\frac{v+s_M}{1+vs_M}=1-\frac{2\pi wn_*^2(1-s^2)}{b^2n^4}\frac{1-v_*}{1+v_*}.\qquad (A1.10)$$

The last equality in this expression is obtained for the high-σ case. Let us consider the cold plasma, $w=mn$. Then Eq.(A1.10) yields

$$x=\left[1-\frac{1}{2\sigma_*}\frac{1-v_*}{1+v_*}\left(\frac{n_*}{n}\right)^3\right]t+f(n),\qquad (A1.11)$$

where $\sigma_*\equiv 4\pi m/(b^2n_*)$. The function $f(n)$ is determined by the initial waveform. For the initially sinusoidal wave,

$$B=B_*\left(1+\alpha\cos\omega x\right),$$

one gets from Eqs(A1.2, A1.9):

$$f(n)=\frac{1}{\omega}\arccos\left\{\frac{1+v_*}{2\alpha}\left[\left(\frac{n}{n_*}\right)^2-1\right]\right\}.\qquad (A1.12)$$

Note that the dimensionless amplitude, α, does not exceed the value $\alpha_{max}=(1+v_*)/2$; $n\to 0$ at some phase of the wave period as $\alpha\to\alpha_{max}$.

According to Eq.(A1.10), any portion of the wave advances with a constant velocity depending only on the plasma density. The compressive parts overtake the expansive ones and eventually a shock arises where the waveform becomes vertical,

$$\frac{dx}{dn}=\frac{d^2x}{dn^2}=0.$$

This occurs at a time

$$t_0=\sqrt{\frac{10}{3}}\frac{2\sigma_*}{9\omega}\frac{1+v_*}{1-v_*}\frac{\left(4-\sqrt{1+15(\alpha/\alpha_{max})^2}\right)^2}{\sqrt{1+15(\alpha/\alpha_{max})^2}-1}.\qquad (A1.12)$$

When the wave amplitude approaches the maximal one, the shock formation time goes to zero; at small amplitudes, t_0 grows as $1/\alpha$ (Fig. 1). At intermediate amplitudes, the shock formation time is $t_0\sim\sigma_*/\omega$ in the frame moving with the plasma, $v_*=0$; if the plasma moves with respect to the observer with an ultrarelativistic velocity, $\gamma_*\gg 1$, the shock formation time is $t_0\sim\sigma_*\gamma_*^2/\omega$.

Now let us consider the radial flow. In this case the governing equations are written as

$$\frac{\partial}{\partial t}\gamma n+\frac{1}{R^2}\frac{\partial}{\partial R}\gamma R^2nv=0;\qquad (A1.13)$$

$$\frac{\partial B}{\partial t}+\frac{1}{R}\frac{\partial}{\partial R}RvB=0;\qquad (A1.14)$$

$$\frac{\partial}{\partial t}T_{00}+\frac{1}{R^2}\frac{\partial}{\partial R}R^2T_{01}=0.\qquad (A1.15)$$

It follows from Eqs.(A1.13, A1.14) that one can present the magnetic field as

$$B=bRn\gamma,\qquad (A1.16)$$

where $b=const$. If the flow is cold, $w=nm$, Eqs.(A1.13, A1.15, A1.16) reduce to Eqs. (A1.1, A1.2, A1.3) by substitution $\widetilde{n}=nR^2$. Therefore the above estimate of the shock formation time remains valid also for the radial flow.

Appendix 2. Energy and momentum of the wave

Let us show that for small amplitude waves, one can separate the energy and momentum of the wave and the flow without

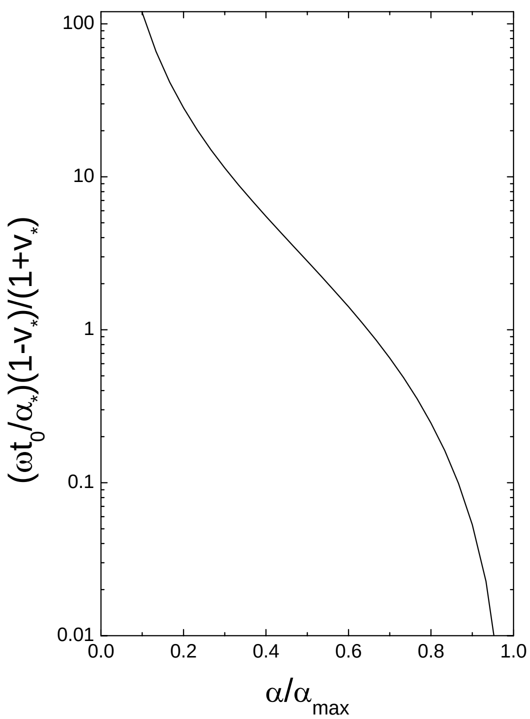

Figure 1. Dependence of the shock formation time, t_0, on the wave amplitude, α.

considering specific waveforms. Let us define the average plasma density, n_*, and velocity, v_*, by the relations

$$< nv\gamma >= n_* v_* \gamma_*; \qquad (A2.1)$$

$$< n\gamma >= n_* \gamma_*, \qquad (A2.2)$$

where the angular brackets denote averaging over the wave period. With such a definition, there is no mass flux associated with the wave.

The total energy and momentum densities are (see Eq.(A1.4-A1.6))

$$T_{00} = \mathcal{W}\gamma^2 - \mathcal{P}; \qquad (A2.3)$$

$$T_{01} = \mathcal{W}v\gamma^2. \qquad (A2.4)$$

Averaging this values, one can present them as a superposition of a flow part dependent only on average plasma parameters and a wave part dependent on the wave amplitude. Calculations are simplified in the frame where plasma is at rest in average, $v_* = 0$.

In a small amplitude wave, the energy and momentum are of the second order in the wave amplitude therefore linear relations, like (see Eq.(A1.8))

$$\frac{\delta n}{n_*} = \frac{\delta v}{s_M},$$

may be used only in the second order terms. The average of the first order terms may be expressed via the second order terms expanding Eqs.(A2.1, A2.2) in δv and δn to the second order. The result is

$$< \delta n >= -\frac{1}{2} n_* < (\delta v)^2 >;$$

$$< \delta v >= -\frac{1}{s_M} < (\delta v)^2 > .$$

Now expanding Eqs.(A2.3, A2.4) and making use of the thermodynamical expression $d\mathcal{E}/dn = \mathcal{W}/n$, (because the wave is isentropic, all thermodynamical values may be considered as functions of n), one yields

$$< T_{00} >= \mathcal{E}_* + \mathcal{W}_* < (\delta v)^2 >;$$

$$< T_{01} >= s_M \mathcal{W}_* < (\delta v)^2 >,$$

where $\mathcal{E}_* \equiv \mathcal{E}(n_*)$ etc. So the energy density is separated into a part which depends only on the average medium density and the wave part which is proportional to the wave amplitude squared. The momentum density of the medium is zero in the proper plasma frame therefore only the wave momentum contributes to $< T_{01} >$. The energy and momentum of the wave coincide, in the proper plasma frame, to within $1 - s_M \sim 1/\sigma$. Making the Lorentz transform, one can get components of the energy-momentum tensor in an arbitrary frame of reference.